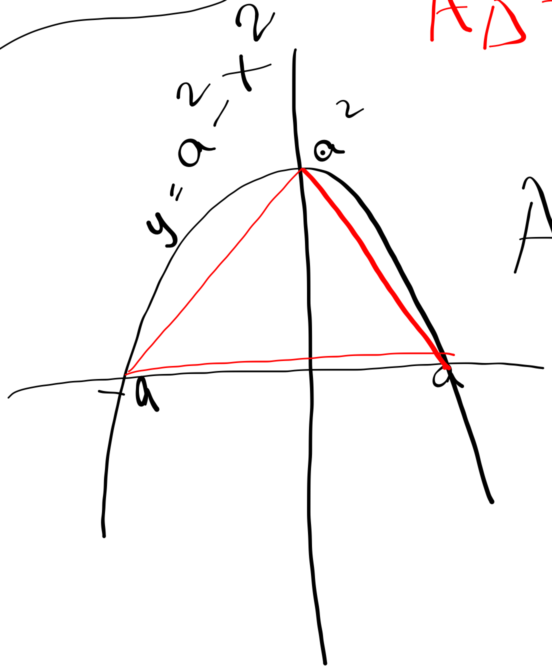
5.5/59)

day 80

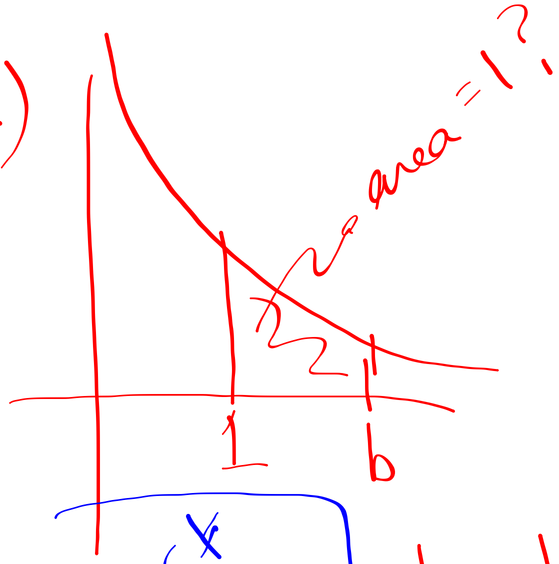
$$A_\Delta = \frac{1}{2}bh = \frac{1}{2}(2a)(a^2)$$
$$= a^3$$



$$A_{\text{enclosed region}} = \int_{-a}^{a} a^2 - x^2\,dx = \left(a^2x - \frac{x^3}{3}\right)\Big|_{-a}^{a}$$
$$= \left(a^3 - \frac{a^3}{3}\right) - \left(-a^3 - \left(\frac{-a^3}{3}\right)\right)$$
$$= 2\left(\frac{2}{3}a^3\right) = \frac{4}{3}a^3$$

5.5/62) $y = \frac{1}{x}$; $x \geq 1$

a)

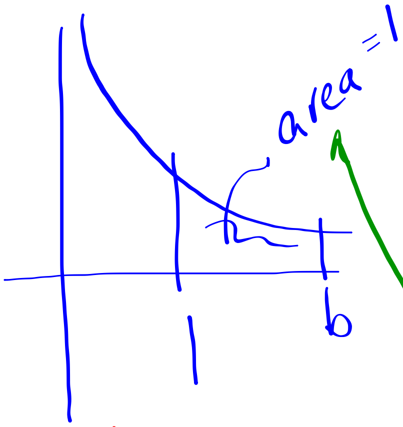


$$\int_1^b \frac{1}{x}\,dx = \ln x \Big|_1^b$$
$$= \ln b - \underbrace{\ln 1}_{0}$$
$$= \ln b$$

when does $\ln b = 1$?
when $b = e^1 = e$!

$$\boxed{\ln x = \int_1^x \frac{1}{t}\,dt}$$

b) $y = \frac{1}{x^p}$; $x \geq 1$, $p < 2$, p rational $\neq 1$



$$\int_1^b x^{-p} = \frac{x^{-p+1}}{-p+1}\Big|_1^b$$
$$= \left(\frac{b^{-p+1}}{-p+1} - \frac{1^{-p+1}}{-p+1}\right) = (1)(1-p)$$

$$\left(b^{-p+1}\right) = -p+1\,(+1) = 2-p$$

$$\left(b^{1-p}\right)^{\frac{1}{1-p}} = b^1 = b$$

$$b = (2-p)^{\frac{1}{1-p}}$$

5.5/37

$f(x) = 1 - \frac{x^2}{a^2}$ over $(0, a)$

avg-value of $f(x)$ = $\frac{\text{Area Under } f(x)}{\text{length of interval}} = \frac{1}{a-0}\int_0^a 1 - \frac{x^2}{a^2}\,dx$

$\frac{1}{a}\left(x - \frac{x^3}{3a^2}\right)\Big|_0^a = \frac{1}{a}\left(\left(a - \frac{a}{3}\right) - (0)\right) = \frac{\frac{2}{3}a}{a} = \frac{2}{3}$

find x

$1 - \frac{x^2}{a^2} = \frac{2}{3}$

$-\frac{x^2}{a^2} = -\frac{1}{3}$

$3x^2 = a$

$x^2 = \sqrt{\frac{a}{3}}$

$a^3 = a^2 - \frac{2}{3}a^3$

$\frac{5}{3}a^3 = a^2$

$a = \frac{3}{5}$

Jan 13-8:44 AM